

To appear in the *International Journal of Systems Science*

Fuzzy Distributed Cooperative Tracking For A Swarm Of Unmanned Aerial Vehicles With Heterogeneous Goals

Georgios P. Kladis[‡], Prathyush P. Menon[§] and Christopher Edwards*

[‡]*KIOS Research Center for Intelligent Systems and Networks, Department of Electrical & Computer Engineering, University of Cyprus, Nicosia, 1678, Cyprus;* [§], ^{*}*Centre for Systems, Dynamics & Control, University of Exeter, The Queen's Dr, Exeter, Devon, EX4 4QJ, United Kingdom*

(Received 00 Month 20XX; Final version received 00 Month 20XX)

This article proposes a systematic analysis for a tracking problem which ensures cooperation amongst a swarm of UAVs, modelled as nonlinear systems with linear and angular velocity constraints, in order to achieve different goals. A distributed Takagi-Sugeno (TS) framework design is adopted for the representation of the nonlinear model of the dynamics of the UAVs. The distributed control law which is introduced is composed of both node and network level information. Firstly feedback gains are synthesised using a Parallel Distributed Compensation (PDC) control law structure, for a collection of isolated UAVs; ignoring communications among the swarm. Then secondly, based on an alternation-like procedure, the resulting feedback gains are used to determine Lyapunov matrices which are utilised at network level to incorporate into the control law the relative differences in the states of the vehicles, and to induce cooperative behaviour. Eventually stability is guaranteed for the entire swarm. The control synthesis is performed using tools from linear control theory: in particular the design criteria are posed as Linear Matrix Inequalities (LMIs). An example based on a UAV tracking scenario is included to outline the efficacy of the approach.

Keywords: Graph theory; Multi-agent systems; Consensus; Parallel Distributed Compensation; Takagi-Sugeno fuzzy model; Linear Matrix Inequalities

1. Introduction

During the last two decades there has been increasing interest in cooperative control which can benefit swarm-based Unmanned Aerial Vehicle (UAV) missions in many ways. The most common problems that have been studied often relate to the stabilisation or tracking of a network of vehicles to introduce cooperative behaviour. In order to achieve this, an important facet is the propagation of information among the vehicles which influences the overall dynamical behaviour of the network.

Multi-agent systems can be represented accurately by nonlinear models in a large domain of operation. However for nonlinear system representations, coupled with the dimensionality of the network, the task of designing a control law is far from trivial. Perhaps not surprisingly most of the existing work has focused on the interconnection of systems with linear dynamics. For example, consensus was examined for multi-agent systems with general linear dynamics in Li et al. (2007), Seo et al. (2009). In Ren and Beard (2008), Sun and Long (2009) consensus problems for agents with single/double or higher integrator dynamics were studied. In Fax and Murray (2004), the

Corresponding authors, Email:

[‡]kladis.georgios@ucy.ac.cy (G.P. Kladis)

[§]p.m.prathyush@exeter.ac.uk (P.P. Menon)

^{*}c.edwards@exeter.ac.uk (C. Edwards)

authors focused on the stabilisation of a network of identical agents with linear dynamics whilst in Cai et al. (2011) the authors suggested necessary and sufficient conditions for swarm stability of high-order linear time-invariant (LTI) multi-agent systems. Swarm-stabilisation problems for high-order LTI singular multi-agent systems with homogeneous agents was investigated in Xi et al. (2013). A distributed output regulation approach for cooperative control of linear multi-agent systems in the presence of communication delays was subsequently presented in Yu and Wang (2014). Additionally, cooperative consensus and pinning control for identical linear time-invariant agent dynamics was investigated in Movric and Lewis (2014). In Liu and Jiang (2013) the authors investigated a cyclic small gain approach to distributed output feedback control of nonlinear multi-agent systems to guarantee the convergence of the agent's outputs to a time-invariant agreement value. In a Leader-Follower setting, consensus was investigated for multi-agent systems with linear/first/higher order dynamics in Wang et al. (2014), Hong et al. (2006) and Liu et al. (2015). However these methodologies often require special assumptions in order to be applied to the entire network, and can often lead to conservative results.

In this paper a general class of networked heterogeneous nonlinear systems is considered. Unlike the previous methodologies which consider linear models to represent the vehicles' motion, in this work, a nonlinear representation of the dynamics of a group of UAV systems, with constraints on angular and linear velocity, is investigated. Contrary to Movric and Lewis (2014) and Liu and Jiang (2013), the proposed methodology considers a systematic analysis for the tracking problem in a more general class of nonlinear systems that can be represented in Takagi-Sugeno (TS) form (Takagi and Sugeno (1985)), in a substitution in which the reference trajectory is time varying. Motivated by the work described in Menon and Edwards (2009), a systematic analysis was suggested in references Kladis et al. (2011a) and Kladis et al. (2011b) for the design of control laws in the TS framework. The synthesis of the controller in Kladis et al. (2011a) and Kladis et al. (2011b) involves a two step procedure, which is enforced locally at node level in a decentralised manner, and produces cooperative behaviour in the network as a whole. The limitation in Kladis et al. (2011b) is that the reference trajectory is common to all vehicles, which may not accommodate realistic real-world scenarios. In this paper, the TS design methodology from Kladis et al. (2011b) is adopted for the solution of the problem of stabilisation/tracking for a swarm of heterogeneous nonlinear systems.

The TS fuzzy model (Takagi and Sugeno (1985)) can represent, with reduced mathematical complexity, a large class of nonlinear systems (Tanaka and Wang (2001)), and due to the structure of the TS system, which is a fuzzy *blending* of linear local models, a systematic approach for proving stability is available. Interesting work addressing the design aspects for TS controllers exists in the literature: see for example, Tanaka and Wang (2001), Guerra et al. (2006), and Lendek et al. (2010).

Specifically, in this paper, the model of the error dynamics of the UAV, as developed in Klančar and Skrjanc (2007) and Kladis et al. (2011), is employed. In the first step of the procedure, the error dynamics of a set of heterogeneous single UAV systems is considered, and a node level control law is designed ignoring interconnections. The node level control law utilises a Parallel Distributed Compensation (PDC) structure as suggested in Wang et al. (1995) and the feedback gains are synthesised, subject to certain design criteria, in the form of Linear Matrix Inequalities (LMIs). Thereafter, via an intermediate step, utilising the resulting feedback gains, less conservative conditions are introduced for the synthesis of the Lyapunov matrices, which are used at a later step. Subsequently, in the second step, dependencies and interconnections among the UAVs are considered and a distributed control law is introduced which is shown to guarantee stability for the entire swarm.

The novelty of this work is that it proposes a methodology for the analysis of a network of heterogeneous nonlinear systems. An intermediate step (the creation of an equivalent TS representation form) allows “decoupling” of the network into node level dynamics. This structure facilitates a systematic analysis using Lyapunov theory for stabilisation/tracking. Eventually it is shown that the resulting Lyapunov matrices, arising from node level analysis, can be used to create a Lyapunov function at the network level. The benefit of the proposed approach is that the analysis and design

is performed at a node level, and thus the problem of stabilisation/tracking is decoupled from the network's large scale topology. Furthermore the methodology can be applied to a reasonably large class of nonlinear systems.

The remainder of the paper is structured as follows: in Section 2.1 the notation is introduced and in Section 2.2 the graph theory tools which are used, and their relevance to a network of systems, are presented. In Sections 2.3 and 2.4, the nonlinear model under investigation and the equivalent TS model are described. Thereafter in Section 3 the architecture of the controller and the LMI conditions to stabilise the system at node (Subsection 3.1) and network (Subsection 3.2) level are described. A swarm based UAV tracking example is included in Section 4, demonstrating the proposed analysis. In Section 5 concluding remarks and possible future research directions are stated.

2. Preliminaries

2.1. Notation

The following notation is used throughout this work. The expression $\mathbf{A} > 0$ denotes that the \mathbf{A} matrix, of appropriate dimensions, is symmetric positive definite. \mathbf{A}^T and \mathbf{A}^{-1} correspond to the transpose and inverse of any square matrix \mathbf{A} . The notation $\text{diag}\{\mathbf{A}_1, \dots, \mathbf{A}_N\}$ represents a block diagonal matrix with diagonal elements \mathbf{A}_i for $i = 1, \dots, N$ and $\mathbf{u}(t) = \text{col}(u^1(t), \dots, u^N(t))$ represents the concatenation of the vectors $u^i(t)$. The symbol $\|\cdot\|$ corresponds to the spectral norm of a matrix, $|\cdot|$ the length of a vector, and \otimes corresponds to the Kronecker product notation. Finally, $\text{sinc}(\theta(t))$ is equal to $\sin(\theta(t))/\theta(t)$ for $\theta(t) \neq 0$.

2.2. Graph theory for a swarm of UAV systems

In this section graph theory preliminaries and their relevance with respect to modelling a swarm of UAV systems is described. Adopting the notation in Royle and Godsil (2001), a graph G is an ordered pair (V, E) , where V is the set of vertices or nodes ($V = \{1, \dots, N\}$) and E is the set of edges, ($E = \{c_1, \dots, c_l\}$), which represent every possible connection between a pair of nodes. In this work a node coincides with a UAV system within the swarm, and the set E denotes the communication links between UAV systems j and i . A graph G can be represented in the form of the adjacency matrix $\mathbf{A}(G) = [\alpha_{ij}] \in \mathbb{R}^{N \times N}$ and is defined by:

$$\alpha_{ij} = \begin{cases} 1, & \forall (i, j) \in E \text{ and } i \neq j \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

For bidirectional graphs, $\alpha_{ij} = \alpha_{ji}$ and the adjacency matrix is symmetric. The degree $\mathbf{D}(G) = [d_{ii}] \in \mathbb{R}^{N \times N}$ of a graph is a diagonal matrix for which $d_{ii} = \sum_{j=1}^N \alpha_{ij}$ and $d_{ij} = 0$, $\forall i \neq j$. The Laplacian of a graph $\mathbf{L}(G) = [\ell_{ij}] \in \mathbb{R}^{N \times N}$ is equal to:

$$\mathbf{L}(G) = \mathbf{D}(G) - \mathbf{A}(G) = [\ell_{ij}] = \begin{cases} \sum_{j=1}^N \alpha_{ij}, & i = j \\ -\alpha_{ij}, & i \neq j \end{cases} \quad (2)$$

According to Royle and Godsil (2001), for undirected (bidirectional) graphs, the Laplacian matrix is symmetric positive semi-definite and satisfies $\sum_{j=1}^N \ell_{ij} = 0$, $\forall i \in V$. In this paper the swarm's communication topology is assumed bidirectional and static.

2.3. UAV model and the tracking problem

The kinematics of the i^{th} ($i = 1, \dots, N$, where N the number of UAVs within the swarm) point-mass UAV satisfies (3), under the assumptions that the UAV is moving in 2D, the thrust and velocity vector are collinear, and there is no slip in the lateral direction. The equations are given

by

$$\begin{aligned}\dot{x}_c^i(t) &= v_{er}^i \cos \theta_c^i(t) \\ \dot{y}_c^i(t) &= v_{er}^i \sin \theta_c^i(t) \\ \dot{\theta}_c^i(t) &= w_{er}^i\end{aligned}\tag{3}$$

where the control inputs w_{er}^i and v_{er}^i , are the linear and angular velocity, respectively. $P_c^i(x_c^i, y_c^i, \theta_c^i)$ will be used to represent the current posture of the UAV with x_c^i, y_c^i the position coordinates, and θ_c^i the heading angle. As in Klančar and Skrjanc (2007) and Kladis et al. (2011), for the purpose of tracking, the error in posture is utilised in this work. In particular, the error posture model of a vehicle is generated with the aid of the reference $P_{ref}^i(x_{ref}^i, y_{ref}^i, \theta_{ref}^i)$ and the current posture $P_c^i(x_c^i, y_c^i, \theta_c^i)$ utilising the kinematics in (3). It should be noted that in this work each vehicle is required to track a unique path $P_{ref}^i(\cdot)$.

According to Nelson and Cox (1988), the tracking error for each UAV in its body axis is governed by:

$$e^i(t) = \begin{bmatrix} \cos(\theta_c^i(t)) & \sin(\theta_c^i(t)) & 0 \\ -\sin(\theta_c^i(t)) & \cos(\theta_c^i(t)) & 0 \\ 0 & 0 & 1 \end{bmatrix} (P_{ref}^i(t) - P_c^i(t))\tag{4}$$

where $e^i(t) = \text{col}(x_e^i(t), y_e^i(t), \theta_e^i(t))$ is the tracking error in the state for the i^{th} UAV. The components represent errors in the $x-y$ plane and heading direction, respectively. As in Klančar and Skrjanc (2007) $u_{er}^i = u_F^i(t) + u^i(t)$ is defined as the control input where $u_F^i(t) = [v_{ref}^i \cos(\theta_e^i(t)), w_{ref}^i]^T$ is the feedforward control action vector, and $u^i(t) = \text{col}(u_v^i, u_w^i)$ is the feedback control action vector. Here $v_{ref}^i, u_v^i(t)$ are the reference and current linear velocities, and $w_{ref}^i, u_w^i(t)$ are the reference and current angular velocities. Following the approach in Section 3.1 of Kladis et al. (2011), taking the time derivative of (4), due to the nonholonomic constraint (i.e. $\dot{x}_{ref}^i \sin(\theta_{ref}^i) = \dot{y}_{ref}^i \cos(\theta_{ref}^i)$), the error dynamics satisfy:

$$\dot{e}^i(t) = f_i(e^i(t)) + g_i(e^i(t))u^i(t)\tag{5}$$

where

$$f_i(e^i(t)) = \begin{bmatrix} 0 & w_{ref}^i & 0 \\ -w_{ref}^i & 0 & v_{ref}^i \text{sinc}(\theta_e^i(t)) \\ 0 & 0 & 0 \end{bmatrix}, \quad g_i(e^i(t)) = \begin{bmatrix} -1 & y_e^i(t) \\ 0 & -x_e^i(t) \\ 0 & -1 \end{bmatrix}\tag{6}$$

Employing the analysis shown in Kladis et al. (2011b), the dynamics in (5) can be transformed into an equivalent TS representation (Takagi and Sugeno (1985)). The task is to design a control law $u^i(t) = \text{col}(u_v^i(t), u_w^i(t))$ for the nonlinear system (5), such that $e^i(t) \rightarrow 0$ as $t \rightarrow \infty$.

2.4. Equivalent Takagi-Sugeno (TS) representation

In this section the TS fuzzy model and the derivation of the equivalent system (5) in TS form are shown. Consider a group of nonidentical UAV systems $i = 1, \dots, N$ described by (5), where $e^i(t) = \text{col}(x_e^i(t), y_e^i(t), \theta_e^i(t))$, $u^i(t) = \text{col}(u_v^i(t), u_w^i(t))$, are the state and input vector respectively. The nonlinear model in (5) can be represented in a compact region of the state-space $\mathcal{X} \subseteq \mathbb{R}^3$ by a TS fuzzy model utilising the sector nonlinearity approach of Kawamoto et al. (1992).

Adopting the notation in Tanaka and Wang (2001), for vehicle i , the TS fuzzy model is formed by $\kappa = 1, \dots, r$ local linear subsystems, where the number of rules r is determined according to the length of the chosen premise vector z , and is equal to $r = 2^{|z|}$. In particular, here $z^i(t) = \text{col}(z_1^i(t), \dots, z_4^i(t))$ is a known premise vector, which may depend on the state vector. Where the premise variables for the model illustrated in (5) are chosen as $z_1^i(t) = w_{ref}^i$, $z_2^i(t) = v_{ref}^i \text{sinc}(\theta_e^i(t))$, $z_3^i(t) = y_e^i(t)$ and $z_4^i(t) = x_e^i(t)$. The premise variables are required to be bounded and the bounds need to be a priori defined in order for the TS to exactly represent the nonlinear model considered (Tanaka and Wang (2001)). Here, for simplicity, it is assumed that $z_1^i(t) \in [a_{1 \min}^i, a_{1 \max}^i]$, $z_2^i(t) \in [a_{2 \min}^i, a_{2 \max}^i]$, $z_3^i(t) \in [a_{3 \min}^i, a_{3 \max}^i]$ and $z_4^i(t) \in [a_{4 \min}^i, a_{4 \max}^i]$. In particular, it is assumed

in this work that, $x_e^i(t), y_e^i(t), \in [-10, 10]$, and $\theta_e^i(t) \in [-\pi/4, \pi/4]$, while z_1^i and z_2^i depend on the reference linear and angular velocities.

Remark 1. The latter bounds on the state are selected according to the specifications of the UAV in order not to lose controllability of the system. These are a priori defined and guarantee the TS exactly represents the nonlinear model considered in Tanaka and Wang (2001). It should be noted that the choice of a different set of bounds may yield a different response of the system.

The TS is represented by implications of **IF – THEN** form or Input-Output form. Since $q = 4$, the number of rules in the fuzzy system is equal to $r = 16$. For the i^{th} system, the general layout for the κ^{th} model rule according to Tanaka and Wang (2001) is:

$$\boxed{\text{IF } z_1^i(t) \text{ is } \Pi_{\kappa\mu}^i \text{ AND } z_2^i(t) \text{ is } \Omega_{\kappa\mu}^i \text{ AND } z_3^i(t) \text{ is } \Psi_{\kappa\mu}^i \text{ AND } z_4^i(t) \text{ is } \Theta_{\kappa\mu}^i \text{ THEN } \dot{e}^i(t) = \mathbf{A}_{\kappa}^i e^i(t) + \mathbf{B}_{\kappa}^i u^i(t)}$$

where the symbols $\Pi_{\kappa\mu}^i(z_1^i(t))$, $\Omega_{\kappa\mu}^i(z_2^i(t))$, $\Psi_{\kappa\mu}^i(z_3^i(t))$, and $\Theta_{\kappa\mu}^i(z_4^i(t))$ denote the fuzzy sets and are normalised to unity. The fuzzy sets are generated utilising the sector nonlinearity approach from Kawamoto et al. (1992). In the model rules, $\mathbf{A}_{\kappa}^i \in \mathbb{R}^{3 \times 3}$, $\mathbf{B}_{\kappa}^i \in \mathbb{R}^{3 \times 2}$, are constant matrices.

In Input-output form, the defuzzification process of the model's rules can be represented by the following polytopic form:

$$\dot{e}^i(t) = \sum_{\kappa=1}^r \lambda_{\kappa}(z^i(t)) [\mathbf{A}_{\kappa}^i e^i(t) + \mathbf{B}_{\kappa}^i u^i(t)] \quad (7)$$

where $r = 16$, the $\lambda_{\kappa}(z^i(t))$ are normalised weighting functions defined by:

$$\lambda_{\kappa}(z^i(t)) = (\prod_{\mu=1}^q M_{\kappa\mu}^i(z_{\mu}^i(t))) / (\sum_{\kappa=1}^r \prod_{\mu=1}^q M_{\kappa\mu}^i(z_{\mu}^i(t))) \quad (8)$$

In (8) $M_{\kappa\mu}^i(z_{\mu}^i(t)) = \{\Pi_{\kappa\mu}^i(z_1^i(t)), \Omega_{\kappa\mu}^i(z_2^i(t)), \Psi_{\kappa\mu}^i(z_3^i(t)), \Theta_{\kappa\mu}^i(z_4^i(t))\}$ is the *truth* value of $z_{\mu}^i(t)$ in the fuzzy set $M_{\kappa\mu}^i$. The weighting terms $\lambda_{\kappa}(z^i(t))$ satisfy the convex sum property for all t (i.e. $\sum_{\kappa=1}^r \lambda_{\kappa}(z^i(t)) = 1$). From the path planning phase, N trajectories are generated for each vehicle to follow, and each TS model (7) represents a nonlinear model in (5).

3. Control law description and swarm tracking

In this section a control law is proposed for the stabilisation of the system in (7). Adopting the notation in Kladis et al. (2011b), the control law is composite and has the form

$$u^i(t) = u_{\tau}(e^i(t)) + \gamma \bar{\mathbf{F}}^i \sum_{j \neq i}^N \alpha_{ij} (e^i(t) - e^j(t)), \quad i = 1, \dots, N \quad (9)$$

where the positive scalar γ defines the level of contribution of the coupling term to the control law $u^i(t)$, and the $\bar{\mathbf{F}}^i \in \mathbb{R}^{2 \times 3}$ are feedback gains associated with the network level components. In (9) the term $u_{\tau}(e^i(t))$ represents the node level component, and is responsible for stabilising the i^{th} system in (7), while the network level component aims to create co-operative behaviour among the vehicles. The node level control component $u_{\tau}(e^i(t))$ will be designed based on TS concepts and has the form referred to in the literature as PDC (Wang et al. (1995)). The task is to create co-operative behaviour within the entire network of N vehicles. Following the methodology in Kladis et al. (2011a) and Kladis et al. (2011b), the control law design is accomplished in two steps for the equivalent TS fuzzy model in (7) of the nonlinear system in (5). In the first step individual vehicles are isolated from the swarm and the interconnections are ignored in the control law (9). Using Lyapunov theory tools the control law for the closed loop node level system is synthesised using an alternation-like procedure. Thereafter, in the second step (network level analysis), the synthesised control law in (9) is used (including the interconnections among vehicles) and it is shown that the entire swarm is stable.

3.1. Step (A): Node level Tracking analysis

In the first step of the analysis, ignoring interconnections, the PDC controller $u_\tau(e^i(t))$ applied to the i^{th} system in (7), is equal to:

$$u_\tau(e^i(t)) = - \sum_{\kappa=1}^r \lambda_\kappa(z^i(t)) \mathbf{F}_\kappa^i e^i(t) \quad (10)$$

for $\kappa = 1, \dots, r$, where the $\mathbf{F}_\kappa^i \in \mathbb{R}^{2 \times 3}$ are the feedback gains. By substitution of the control law (10) into (7), the node level closed-loop error dynamics are equal to:

$$\dot{e}^i(t) = \sum_{\kappa=1}^r \sum_{\mu=1}^r \lambda_\kappa(z^i(t)) \lambda_\mu(z^i(t)) \mathbb{A}_{\kappa\mu}^i e^i(t) \quad (11)$$

where $\mathbb{A}_{\kappa\mu}^i = \mathbf{A}_\kappa^i - \mathbf{B}_\kappa^i \mathbf{F}_\mu^i$. According to Wang et al. (1996) the dynamics in (11) expand to:

$$\dot{e}^i(t) = \sum_{\kappa=1}^r \lambda_\kappa(z^i(t))^2 \mathbb{A}_{\kappa\kappa}^i e^i(t) + 2 \sum_{\kappa=1}^r \sum_{\mu < \kappa}^r \lambda_\kappa(z^i(t)) \lambda_\mu(z^i(t)) \left(\frac{\mathbb{A}_{\kappa\mu}^i + \mathbb{A}_{\mu\kappa}^i}{2} \right) e^i(t) \quad (12)$$

The task is to determine the feedback gains \mathbf{F}_κ^i , and a symmetric positive definite matrix $\mathbf{P}^i \in \mathbb{R}^{3 \times 3}$, such that a local performance criteria for stability is satisfied. Here $\mathbf{P}^i \in \mathbb{R}^{3 \times 3}$ is assumed to have the form:

$$\mathbf{P}^i = \begin{bmatrix} P_1 & P_2 \\ P_2^T & \mathbf{P}_3^i \end{bmatrix}, \quad i = 1, \dots, N \quad (13)$$

where P_1 is a scalar, $P_2 \in \mathbb{R}^{2 \times 1}$, and $\mathbf{P}_3^i \in \mathbb{R}^{2 \times 2}$. Consider a potential Lyapunov function

$$v^i(t) = e^i(t)^T \mathbf{P}^i e^i(t) \quad (14)$$

for $i = 1, \dots, N$ where \mathbf{P}^i has the structure in (13). The objective is to ensure the derivatives $\dot{v}^i(t)$ are negative definite for all $e^i(t) \neq 0$. Taking the derivative of (14), and using (11), creates a set of Bilinear Matrix Inequalities (BMIs) for the synthesis of the feedback gains \mathbf{F}_κ^i and the Lyapunov matrices \mathbf{P}^i to ensure that the systems at node level are stable. Finding a solution of these BMIs is, if not impossible, certainly not a trivial task. To overcome this problem, and since in this work only sufficient solutions are required, a procedure based on ‘‘alternation’’ is utilised: see for example Goh et al. (1995). In essence, firstly the feedback gains \mathbf{F}_κ^i and a common symmetric positive definite matrix $\mathbf{P} \in \mathbb{R}^{3 \times 3}$ (as in (13) with $\mathbf{P}_3^i = \mathbf{P}_3$) are calculated such that the derivatives of

$$v^i(t) = e^i(t)^T \mathbf{P} e^i(t) \quad (15)$$

are negative definite. Thereafter using the resulting feedback gains \mathbf{F}_κ^i , and P_1, P_2 , the positive definite matrices $\mathbf{P}^i \in \mathbb{R}^{3 \times 3}$ (as in (13)) are calculated such that the derivatives of (14) are negative definite. The latter procedure will prove helpful at a network level analysis.

In the first step of the node level stabilisation process, utilising the closed loop dynamics in (12), the time derivative of the Lyapunov functions (15) is equal to:

$$\begin{aligned} \dot{v}^i(t) = & \sum_{\kappa=1}^r \lambda_\kappa^2(z^i(t)) [e^i(t)^T (\mathbb{A}_{\kappa\kappa}^i{}^T \mathbf{P} + \mathbf{P} \mathbb{A}_{\kappa\kappa}^i) e^i(t)] \\ & \dots + 2 \sum_{\kappa=1}^r \sum_{\mu < \kappa}^r \lambda_\kappa(z^i(t)) \lambda_\mu(z^i(t)) \left[e^i(t)^T \left(\frac{(\mathbb{A}_{\kappa\mu}^i + \mathbb{A}_{\mu\kappa}^i)^T \mathbf{P} + \mathbf{P} (\mathbb{A}_{\kappa\mu}^i + \mathbb{A}_{\mu\kappa}^i)}{2} \right) e^i(t) \right] \end{aligned} \quad (16)$$

Using Theorem 7 (pp.51 in Tanaka and Wang (2001)), the stabilisation of the i^{th} TS system in (16) is ensured via the PDC control law in (10) if there exists a symmetric positive matrix $\mathbf{X} > 0$

($\mathbf{X} \in \mathbb{R}^{3 \times 3}$) and matrices $\Xi_\mu^i \in \mathbb{R}^{2 \times 3}$ for $\mu = 1, \dots, r$ such that the following conditions hold:

$$\begin{cases} \mathbf{X} > 0 \\ \Upsilon_{\kappa\kappa}^i < 0 \\ \Upsilon_{\kappa\mu}^i + \Upsilon_{\mu\kappa}^i \leq 0, \quad \kappa < \mu, \kappa, \mu = 1, \dots, r, \quad \lambda_\kappa(z^i(t)) \cap \lambda_\mu(z^i(t)) \neq \emptyset \end{cases}, \quad \kappa = 1, \dots, r \quad (17)$$

for $i = 1, \dots, N$, where

$$\Upsilon_{\kappa\mu}^i = \mathbf{X} \mathbf{A}_\kappa^{iT} + \mathbf{A}_\kappa^i \mathbf{X} - \Xi_\mu^{iT} \mathbf{B}_\kappa^{iT} - \mathbf{B}_\kappa^i \Xi_\mu^i \quad (18)$$

In (17) letting $\mathbf{X} = \mathbf{P}^{-1}$, and $\Xi_\mu^i = \mathbf{F}_\mu^i \mathbf{X}$, the feedback gains can be recovered as:

$$\mathbf{F}_\mu^i = \Xi_\mu^i \mathbf{X}^{-1} \quad (19)$$

The conditions in (17) can be modified to include performance characteristics: for example by the introduction of a decay rate $\eta > 0$ and constraints on the control effort. The introduction of a decay rate is equivalent to ensuring:

$$\dot{v}^i(t) + 2\eta v^i(t) < 0 \quad (20)$$

and the conditions in (17) are replaced by

$$\begin{cases} \mathbf{X} > 0 \\ \Upsilon_{\kappa\kappa}^i + 2\eta \mathbf{X} < 0 \\ \Upsilon_{\kappa\mu}^i + \Upsilon_{\mu\kappa}^i + 4\eta \mathbf{X} \leq 0, \quad \kappa < \mu, \kappa, \mu = 1, \dots, r, \quad \lambda_\kappa(z^i(t)) \cap \lambda_\mu(z^i(t)) \neq \emptyset \end{cases}, \quad \kappa = 1, \dots, r \quad (21)$$

for $i = 1, \dots, N$, where $\Upsilon_{\kappa\mu}^i$ is equal to (18).

In order to constrain the control effort, a generalised eigenvalue problem subject to (17) or (21), can be used as suggested in Boyd et al. (1994). Provided that the initial conditions are a-priori known, the control effort can be constrained to satisfy $\|u_\tau(e^i(t))\|_2 \leq \nu$ by the optimisation problem:

$$\min_{\mathbf{X}, \Xi_1^i, \dots, \Xi_r^i} \nu \quad (22)$$

subject to the LMIs (17) or (21) and

$$\begin{bmatrix} 1 & e(0)^T \\ e(0) & \mathbf{X} \end{bmatrix} \geq 0 \quad (23)$$

$$\begin{bmatrix} \mathbf{X} & \Xi_\kappa^{iT} \\ \Xi_\kappa^i & \nu^2 \mathbf{I} \end{bmatrix} \geq 0 \quad (24)$$

for $\kappa = 1, \dots, r$. Provided the feedback gains \mathbf{F}_μ^i are chosen for a common Lyapunov matrix \mathbf{P} satisfying conditions (17) (or (21)), and (23), (24), stability can be guaranteed for any set of initial conditions $e^i(0) \in \mathcal{X} \subseteq \mathbb{R}^3$.

Now, at the second step of the node level stabilisation process, the resulting \mathbf{F}_μ^i , P_1 , and P_2 from the solution of the above LMIs is utilised to determine \mathbf{P}^i in (13) such that the derivatives of (14) are negative definite. Using the closed loop dynamics in (12) and taking the time derivative of (14) the block matrices

$$\mathbf{A}_{\kappa\mu}^i = \begin{bmatrix} A_{11\kappa\mu}^i & A_{12\kappa\mu}^i \\ A_{21\kappa\mu}^i & \mathbf{A}_{22\kappa\mu}^i \end{bmatrix}$$

(with $A_{11\kappa\mu}^i$ as scalars, $A_{12\kappa\mu}^i \in \mathbb{R}^{1 \times 2}$, $A_{21\kappa\mu}^i \in \mathbb{R}^{2 \times 1}$, and $\mathbf{A}_{22\kappa\mu}^i \in \mathbb{R}^{2 \times 2}$) are known for the \mathbf{F}_μ^i resulting from the solution of the optimisation problem (22) subject to LMIs (17) (or (21)) and (23), (24). Using Theorem 7 (pp.51 in Tanaka and Wang (2001)), the stabilisation of the i^{th} TS system by the Lyapunov function (14) is ensured via the PDC control law in (10) if there exist symmetric positive matrices $\mathbf{P}^i > 0$ ($\mathbf{P}^i \in \mathbb{R}^{3 \times 3}$ with the special structure in (13)), such that the following conditions hold:

$$\begin{cases} \mathbf{P}^i > 0 \\ \begin{bmatrix} \Upsilon_{11\kappa\kappa}^i & \Upsilon_{12\kappa\kappa}^i \\ \Upsilon_{21\kappa\kappa}^i & \Upsilon_{22\kappa\kappa}^i \end{bmatrix} < 0 \\ \begin{bmatrix} \Upsilon_{11\kappa\mu}^i + \Upsilon_{11\mu\kappa}^i & \Upsilon_{12\kappa\mu}^i + \Upsilon_{12\mu\kappa}^i \\ \Upsilon_{21\kappa\mu}^i + \Upsilon_{21\mu\kappa}^i & \Upsilon_{22\kappa\mu}^i + \Upsilon_{22\mu\kappa}^i \end{bmatrix} \leq 0, \quad \kappa < \mu, \kappa, \mu = 1, \dots, r, \quad \lambda_\kappa(z^i(t)) \cap \lambda_\mu(z^i(t)) \neq \emptyset \end{cases}, \quad \kappa = 1, \dots, r \quad (25)$$

for $i = 1, \dots, N$, where

$$\begin{aligned}\Upsilon_{11\kappa\mu}^i &= A_{11\kappa\mu}^i P_1 + P_1 A_{11\kappa\mu}^{iT} + A_{12\kappa\mu}^i P_2^T + P_2 A_{12\kappa\mu}^{iT} \\ \Upsilon_{12\kappa\mu}^i &= A_{11\kappa\mu}^i P_2 + A_{12\kappa\mu}^i \mathbf{P}_3^i + P_1 A_{21\kappa\mu}^{iT} + P_2 A_{21\kappa\mu}^{iT} \\ \Upsilon_{21\kappa\mu}^i &= A_{21\kappa\mu}^i P_1 + \mathbf{A}_{22\kappa\mu}^i P_2^T + P_2^T A_{11\kappa\mu}^{iT} + \mathbf{P}_3^i A_{12\kappa\mu}^{iT} \\ \Upsilon_{22\kappa\mu}^i &= A_{21\kappa\mu}^i P_2 + P_2^T A_{21\kappa\mu}^{iT} + \mathbf{A}_{22\kappa\mu}^i \mathbf{P}_3^i + \mathbf{P}_3^i \mathbf{A}_{22\kappa\mu}^{iT}\end{aligned}\quad (26)$$

Similarly, the conditions in (25) can be modified to include performance characteristics such as decay rate. Additionally, including the constraints on the control effort and using the resulting ν from the optimisation problem in (22), the stabilisation of the i^{th} TS system using the Lyapunov function (14) is ensured via the PDC control law in (10) if there exist symmetric positive matrices $\mathbf{P}^i > 0$ ($\mathbf{P}^i \in \mathbb{R}^{3 \times 3}$ with the special structure in (13)) such that the conditions in (25) (or the modified ones accounting for the decay rate), and

$$e(0)^T \mathbf{P}^i e(0) - 1 \leq 0 \quad (27a)$$

$$\nu^{-2} \mathbf{F}_\kappa^i T \mathbf{F}_\kappa^i - \mathbf{P}^i \leq 0 \quad (27b)$$

hold for $\kappa = 1, \dots, r$. In (27b) the \mathbf{F}_κ^i are set from the previous procedure. Provided the Lyapunov matrices \mathbf{P}^i are chosen (for the \mathbf{F}_μ^i from the previous step) to satisfy conditions (25), and (27a), (27b), stability can be guaranteed for any set of initial conditions $e^i(0) \in \mathcal{X}$. It should be noted that even in the case when the optimisation problem is infeasible, slack variables can be introduced in the right hand side of the conditions, in order to find sufficient conditions to the problem: see for example Tanaka and Wang (2001) and Liu and Zhang (2003).

Remark 2. Since heterogeneous systems are considered, the number of LMIs involved in the synthesis of the control law grow rapidly for the node level analysis. For the worst case of N different vehicles, $2 \times N$ sets of 154 LMIs are required in order to solve for \mathbf{F}_κ^i , \mathbf{P} and then for \mathbf{P}^i (based on “alternation”). To overcome this computational burden, the conditions can be modified by applying the node level control law with feedback gains \mathbf{F}_μ and a Lyapunov matrix \mathbf{P} , which will reduce the computational complexity, as illustrated in Kladis et al. (2011a) and Kladis et al. (2011b). However the latter leads to more conservative results compared to the conditions presented above. Additionally, common quadratic Lyapunov functions tend to be conservative and may not exist for complex nonlinear systems. To overcome this, it is common practice to use piecewise quadratic Lyapunov functions (see for example articles Qiu et al. (2012), and Qiu et al. (2010)).

Based on the node level stabilisation process, a second step is undertaken as discussed in the next subsection.

3.2. Step (B): Network level Tracking analysis

At a network level, the relative state information among neighboring UAVs is included in the control law $u(e^i(t))$. Utilising the Laplacian matrix defined in (2), the network control law is defined in (9) with $u_\tau(e^i(t))$ equal to (10). Substituting the control law in (9) into system (7), and using the Kronecker product notation (Horn and Johnson (1985)), the network error dynamics can be written in a compact form as:

$$\dot{e}(t) = [\mathcal{A}(z(t)) + \gamma \mathcal{B}(z(t))(\mathbf{L} \otimes \mathbf{I}_n)]e(t) \quad (28)$$

where

$$\mathcal{A}(z(t)) = \text{diag}\left\{\sum_{\kappa=1}^r \sum_{\mu=1}^r \lambda_\kappa(z^1(t)) \lambda_\mu(z^1(t)) \mathbf{A}_{\kappa\mu}^1, \dots, \sum_{\kappa=1}^r \sum_{\mu=1}^r \lambda_\kappa(z^N(t)) \lambda_\mu(z^N(t)) \mathbf{A}_{\kappa\mu}^N\right\} \quad (29)$$

$$\mathcal{B}(z(t)) = \text{diag}\left\{\sum_{\kappa=1}^r \lambda_\kappa(z^1(t)) \mathbf{B}_\kappa^1 \bar{\mathbf{F}}^1, \dots, \sum_{\kappa=1}^r \lambda_\kappa(z^N(t)) \mathbf{B}_\kappa^N \bar{\mathbf{F}}^N\right\} \quad (30)$$

and $e(t)$ is the concatenation of the state vectors $e^i(t)$ so that $e(t) = \text{col}(e^1(t), \dots, e^N(t))$.

Define a candidate Lyapunov function for the swarm as

$$V(t) = \sum_{i=1}^N e^i(t)^T \mathbf{P}^i e^i(t) \quad (31)$$

where the symmetric positive definite matrices \mathbf{P}^i are from the earlier node level synthesis in Subsection 3.1. Taking the time derivative of (31), and substituting for the closed loop network error dynamics yields

$$\dot{V}(t) = V_1 + V_2 \quad (32)$$

where

$$V_1 = \sum_{i=1}^N \sum_{\kappa=1}^r \sum_{\mu=1}^r \lambda_{\kappa}(z^i(t)) \lambda_{\mu}(z^i(t)) e^i(t)^T [\mathbb{A}_{\kappa\mu}^i{}^T \mathbf{P}^i + \mathbf{P}^i \mathbb{A}_{\kappa\mu}^i] e^i(t) \quad (33)$$

$$V_2 = 2\gamma e(t)^T \mathcal{PB}(z(t))(\mathbf{L} \otimes \mathbf{I}_n) e(t) \quad (34)$$

and $\mathcal{P} = \text{diag}\{\mathbf{P}^1, \dots, \mathbf{P}^N\}$. For the swarm of UAVs, where each vehicle is required to track a predefined reference trajectory, it is sufficient to show that $\dot{V}(t) < 0$. Utilising the stabilisation procedure from the first step of the design process in Subsection 3.1, for the choice of Lyapunov matrices \mathbf{P}^i and feedback gains \mathbf{F}_{μ}^i , $V_1 < 0$. Hence all that needs to be shown is that V_2 is negative semi definite for all $e(t) \neq 0$. It is evident from the TS model that the input matrix \mathbf{B}_{κ}^i is time varying because of (6); however the first column is constant: i.e. $\mathbf{B}_{\kappa}^i = [\mathbf{B}_1, \mathbf{B}_{2\kappa}^i]$. Here by design

$$\bar{\mathbf{F}}^i := -[\mathbf{B}_1, 0]^T \mathbf{P}^i \quad (35)$$

with \mathbf{P}^i equal to (13). Using this choice of $\bar{\mathbf{F}}^i$ means that $\mathcal{PB}(z(t)) = -\text{diag}\{\mathbf{P}^1 \mathbf{B}_1 \mathbf{B}_1^T \mathbf{P}^1, \dots, \mathbf{P}^N \mathbf{B}_1 \mathbf{B}_1^T \mathbf{P}^N\} = -(\mathbf{I}_N \otimes \mathbf{P}_a)$. By appropriate substitutions it can be easily shown that $\mathbf{P}_a \geq 0$. As a result of $\bar{\mathbf{F}}^i$, (34) becomes

$$V_2 = -2\gamma e(t)^T (\mathbf{L} \otimes \mathbf{P}_a) e(t) \quad (36)$$

Since the Laplacian \mathbf{L} is positive semi definite and by construction $\mathbf{P}_a \geq 0$, it follows that $-(\mathbf{L} \otimes \mathbf{P}_a) \leq 0$ from Corollary 4.2.13 in Horn and Johnson (1985). Thus (32) is negative definite for all $e(t) \neq 0$ and the error dynamics of the swarm is stable.

4. Simulation example

In this section, a tracking scenario is considered where a swarm of nonidentical UAVs is deployed to collectively follow the prescribed trajectories of a virtual leader from any initial conditions satisfying the bounds on the state space. The derivation of the TS UAV model is performed according to Subsection 2.4. The reference track considered for each vehicle is referred to in the literature as the Dubins path (Dubins (1957)) and is assumed to be a-priori known from the mission planning phase. The reference track comprises line segments and circular arcs. This is constructed via principles from Euclidean Geometry and the design procedure can be found in Shanmugavel (2007). Here it is assumed that each UAV is assigned an unique reference trajectory to follow, with $v_{ref}^i = v_{ref}$.

4.1. Swarm Tracking for heterogeneous UAV error dynamics

For this example 20 UAVs are deployed, starting from a common base, to collectively follow the prescribed trajectories. They are interconnected through control law (9). Every UAV is represented by the TS model described earlier in Subsection 2.4, and has the form of (7). Since the reference trajectories are different for each of the UAVs (see Fig.1, dashed lines) the TS models in (7) are different. A random static planar graph $G(20, 98)$ with the property of algebraic connectivity is considered. The adjacency matrix is constructed satisfying the properties addressed in Section 2.2. The task here is for all vehicles to track the predefined trajectories. Following the procedure introduced in Section 3.1, initially ignoring interconnections among the vehicles at the first step, the LMIs are specified from the error dynamics of the node level systems (11). The minimisation problem subject to the LMI conditions has been solved using YALMIP (Laofberg (2004)) which is

a language for advanced modeling and solution of convex and nonconvex optimisation problems. Solving the minimisation problem in (22) subject to the LMIs in (21) and (23), (24) leads to the feedback gains \mathbf{F}_μ^i , with $\kappa, \mu = 1, \dots, 16$, which are recovered from (19), together with a common positive definite matrix \mathbf{P} . The eigenvalue distribution for $\mathbf{A}_\kappa^i - \mathbf{B}_\kappa^i \mathbf{F}_\mu^i$ and \mathbf{P} , is shown in Fig.3, and denoted by “o”. The different shades of grey correspond to each of the systems i .

From the optimisation problem in (22) the resulting ν is equal to 28.7764, for $\eta = 0.5$ and $e^i(0) = [10, 10, 0.078]^T$ subject to the conditions in (21) and (23), (24). Thereafter the alternation-like procedure is used. In the node level stabilisation process, the feedback gains \mathbf{F}_μ^i , calculated earlier, are used. The objective is to employ less conservative conditions in order to find the Lyapunov matrices \mathbf{P}^i (with the special structure in (13)) for use in the candidate Lyapunov functions (14). For the specific parameters η , $e^i(0)$, and ν , \mathbf{F}_μ^i , respectively, the modified conditions in (25) (with a decay rate), (27a), and (27b) are feasible. The eigenvalues of the resulting Lyapunov matrices \mathbf{P}^i , with $i = 1, \dots, 20$, are depicted in Fig.3 and are denoted by “+”. In \mathbf{P}^i , with the special structure in (13), the numerical values $P_1 = 0.0017$ and $P_2 = [-0.0000 \ -0.0009]$ have been obtained. Then utilising the results from the first step in Section 3.1 it can be verified that the choice of gain $\bar{\mathbf{F}}^i$, given in (35), ensures $\dot{V}(t) < 0$, and the entire network of systems is stable.

Simulating the network in (28) for these gains and using the communication channel in (9) with $\gamma = 0.7$, the error in the state is shown in Fig.3. The overall control law (9) is synthesised and is added to the feed-forward control action vector $u_F^i(t) = [v_{ref} \cos(\theta_e^i(t)), w_{ref}^i]^T$ to generate u_{er}^i (depicted in Fig.4). The control input u_{er}^i is used in the i^{th} UAV model (3). The resulting trajectories and the heading angles of the vehicles converge to the prescribed trajectories as depicted in Figs.1 and 2, respectively. It is noted that since all vehicles are required to have the same reference linear velocity, then they will finish their mission at different time instances. Here it is assumed that as soon as each vehicle follows the prescribed reference track, then it switches control mode to “idle” (ie. with angular velocity equal to zero) in order for the operator to act accordingly.

Contrary to Li et al. (2007), Seo et al. (2009), Ren and Beard (2008), Sun and Long (2009) which consider linear/first/second order models to represent the vehicles’ motion, in this work, a nonlinear representation of the dynamics of a group of UAV systems, with constraints on angular and linear velocity, was investigated. The novelty of this work is that it proposes a methodology for the analysis of a network of heterogeneous nonlinear systems. An intermediate step (the creation of an equivalent TS representation form) allows a “decoupling procedure” for the network into node level dynamics. This structure facilitates a systematic analysis using Lyapunov theory for stabilisation/tracking. As argued above, the resulting Lyapunov matrices, arising from the node level analysis, can be used to create a Lyapunov function at the network level. The benefit of the proposed approach is that the analysis and design is decoupled from the network’s large scale topology. Also the methodology can be applied to a reasonably large class of nonlinear systems.

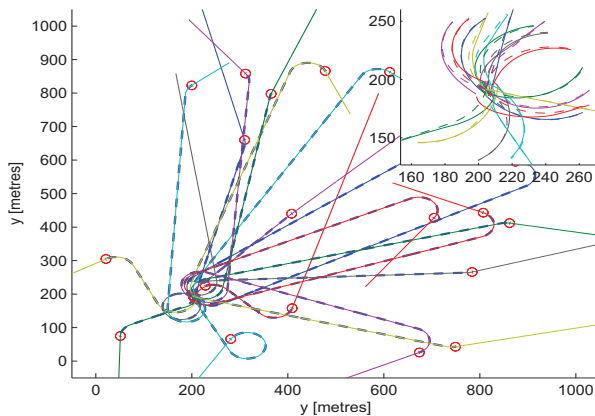


Figure 1. Trajectories of the swarm in $x - y$ plane (solid lines) versus the prescribed trajectories (dashed line).

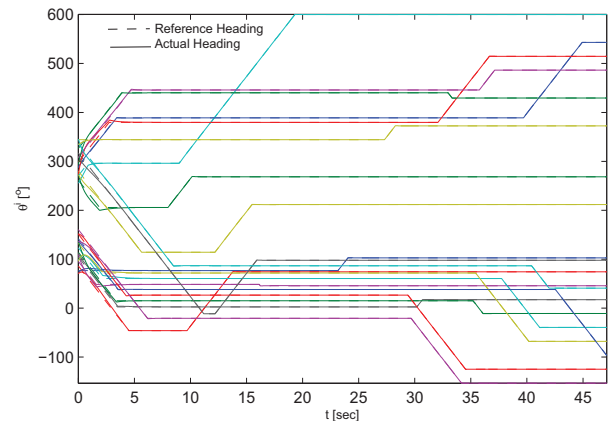


Figure 2. Heading angle profile for each UAV (solid lines) versus heading angle of the prescribed trajectories (dashed line).

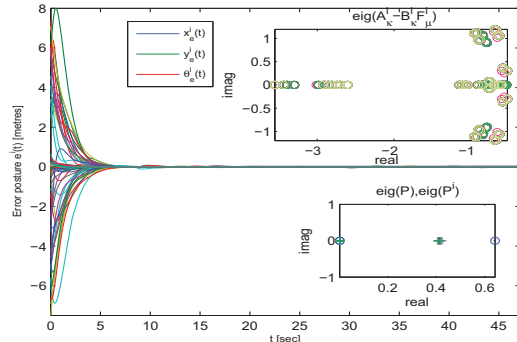


Figure 3. Tracking error $e^i(t) = [x_e^i(t), y_e^i(t), \theta_e^i(t)]^T$ and eigenvalue distribution.

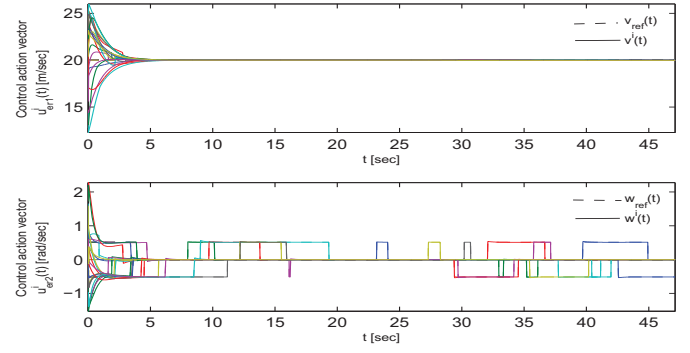


Figure 4. Control action vector $u_{er}^i(t) = [v_{er}^i(t), w_{er}^i(t)]$.

5. Conclusions

This work has proposed a systematic analysis method for tracking problems in a network of non-identical UAVs that can be represented in a polytopic form (TS representation). In particular, the dependencies among the UAVs are represented using graph theory tools. Due to the structure, the properties, and the reduced mathematical complexity of the TS representation, a decoupling of the network into node level dynamics is achieved, which simplifies the stability analysis. It also facilitates the design of a control law for a reasonably general class of nonlinear systems. A two step LMI procedure is suggested for the design of the state feedback controllers. An illustrative example was included to outline the potential of the proposed analysis. The benefit of this approach is that the design of the controller is decoupled from the size of the network and its topology. Additionally, it allows a convenient choice of feedback gains for the network level dynamics, and the methodology can be applied to a reasonably large class of nonlinear systems.

There are at least two directions for potential further work. Firstly, it may worth devising strategies such that less conservative conditions are employed in order to reduce the computational complexity of the LMIs. Secondly, it may be possible to modify the underpinning system such that the reference trajectory is available to a subset of vehicles within the swarm.

Acknowledgement(s)

This work was funded by EPSRC Platform Grant :EP/D029937/1 Title: PLATFORM: CONTROL OF COMPLEX SYSTEMS.

References

- Boyd, S., El Ghaoui, L., Feron, E., and Balakrishnan, V. (1994), "Linear matrix inequalities in system and control theory," *Studies in Applied Mathematics, Philadelphia*.
- Cai, N., Xi, J., and Zhong, Y. (2011), "Swarm Stability of High-Order Linear Time-Invariant Swarm Systems," *IET Control Theory and Applications*, 5, 402–408.
- Dubins, L.E. (1957), "On curves of minimal length with a constraint on average curvature and with prescribed initial and terminal positions and tangent.," *American Journal of Mathematics*, 79, 497–516.
- Fax, J.A., and Murray, R.M. (2004), "Information flow and cooperative control of vehicle formations," *IEEE Transactions on Automatic Control*, 49, 1465–1476.
- Goh, K.C., Safonov, M.G., and Papavassilopoulos, G.P. (1995), "Global optimization for the biaffine matrix inequality problem," *J. Global Opt.*, 7(4), 365–380.
- Guerra, T.M., Kruszewski, A., Vermeiren, L., and Tirmant, H. (2006), "Conditions of output stabilization for nonlinear models in the Takagi - Sugeno form," *Fuzzy Sets and Systems*, 157(9), 1248–1259.
- Hong, Y., Hu, J., and Gao, L. (2006), "Tracking control for multi-agent consensus with an active leader and variable topology," *Automatica*, 42(7), 1177–1182.
- Horn, R.A., and Johnson, C.R., *Topics in Matrix Analysis*, New York: Cambridge U.P. (1985).
- Kawamoto, S., Tada, K., Ishigame, A., and Taniguchi, T. (1992), "An Approach to Stability Analysis of Second Order Fuzzy Systems," *Proceedings of First IEEE Inter. Conf. on Fuzzy Systems*, 1, 1427–1434.

- Kladis, G.P., Economou, J.T., Lauber, J., and Guerra, T.M. (2011), "Energy Conservation Based Fuzzy Tracking For Unmanned Aerial Vehicle Missions Under Priori Known Wind Information.," *In Elsevier Journal of Engineering applications in Artificial intelligence*, 24(2), 278–294.
- Kladis, G.P., Menon, P.P., and Edwards, C. (Denver, USA, 2011.a), "Distributed Fuzzy Tracking For Multi-agent Systems.," *In 2011 IEEE Multi-Conference on Systems and Control (MSC)*.
- Kladis, G.P., Menon, P.P., and Edwards, C. (Orlando, Florida, 2011b), "Cooperative tracking for a swarm of Unmanned Aerial Vehicles: A distributed Tagaki-Sugeno fuzzy framework design," *50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC)*, pp. 4114 – 4119.
- Klančar, G., and Skrjanc, I. (2007), "Tracking-error model-based predictive control for mobile robots in real time," *Robotics and Autonomous Systems*, 55(3), 460–469.
- Laofberg, J. (2004), "YALMIP: A toolbox for modeling and optimization in MATLAB," *In: Proc. CACSD conf.*, p. <http://control.ee.ethz.ch/joloef/yalmip.php>.
- Lendek, Z., Lauber, J., Guerra, T., Babuska, R., and De Schutter, B. (2010), "Adaptive observers for TS fuzzy systems with unknown polynomial inputs," *Fuzzy Sets and Systems*, 161(15), 2043–2065.
- Li, Z.K., Duan, Z.S., and Huang, L. (2007), "Global synchronization of complex Lure networks," *In Proc. Chinese Contr. Conf., Zhangjiajie, China.*, pp. 304–308.
- Liu, H., Cheng, L., Tan, M., Hou, Z., and Wang, Y. (2015), "Distributed exponential finite-time coordination of multi-agent systems: containment control and consensus," *Inter. Journal of Control*, 88(2), 237–247.
- Liu, T., and Jiang, Z.P. (2013), "Distributed Output-Feedback Control of Nonlinear Multi-Agent Systems," *IEEE Transactions on Automatic Control*, 58(11), 2912–2917.
- Liu, X., and Zhang, Q. (2003), "New approaches to controller designs based on fuzzy observers for Takagi - Sugeno fuzzy systems via LMI," *Automatica*, 39(9), 1571–1582.
- Menon, P., and Edwards, C. (2009), "Decentralised static output feedback stabilisation and synchronisation of networks.," *Automatica*, 45(12), 2910–2916.
- Movric, K.H., and Lewis, F.L. (2014), "Cooperative Optimal Control for Multi-Agent Systems on Directed Graph Topologies," *IEEE Transactions on Automatic Control*, 59(3), 769–774.
- Nelson, W., and Cox, L. (1988), "Local Path Control for an Autonomous Vehicles," *Proc. IEEE International Conference on Robotics and Automation*, pp. 1504–1510.
- Qiu, J., Feng, G., and Gao, H. (2010), "Fuzzy-Model-Based Piecewise Hinf Static-Output-Feedback Controller Design for Networked Nonlinear Systems," *IEEE Transactions on Fuzzy Systems*, 18(5), 919–934.
- Qiu, J., Feng, G., and Gao, H. (2012), "Observer-Based Piecewise Affine Output Feedback Controller Synthesis of Continuous-Time TS Fuzzy Affine Dynamic Systems Using Quantized Measurements," *IEEE Transactions on Fuzzy Systems*, 20(6), 1046–1062.
- Ren, W., and Beard, R., *Distributed Consensus in Multi-Vehicle Cooperative Control*, Communications and Control Engineering, London, U.K.: Springer-Verlag (2008).
- Royle, G., and Godsil, C., *Algebraic graph theory*, New York: Springer Verlag (2001).
- Seo, J.H., Shim, H., and Back, J. (2009), "Consensus of high-order linear systems using dynamic output feedback compensator: Low gain approach," *Automatica*, 45(11), 2659–2664.
- Shanmugavel, M. (2007), "Path Planning of Multiple Autonomous Vehicles," Cranfield University UK.
- Sun, Y.G., and Long, W. (2009), "Consensus problems in networks of agents with double-integrator dynamics and time-varying delays," *Int. Journal Control*, 82(10), 1937–1945.
- Takagi, T., and Sugeno, M. (1985), "Fuzzy identification of systems and its application to modeling and control," *IEEE Trans. On System Man and Cybernetics*, 15(1), 116–132.
- Tanaka, K., and Wang, H.O., *Fuzzy control systems design and analysis: A Linear Matrix Inequality Approach*, New York: 2001 John Wiley and Sons, Inc (2001).
- Wang, H.O., Tanaka, K., and Griffin, M.F. (1995), "Parallel Distributed Compensation of Nonlinear Systems by Takagi-Sugeno Fuzzy Model," *In Proc. FUZZ-IEEE/IFES'95*, pp. 531–538.
- Wang, H.O., Tanaka, K., and Takayuki, I. (1996), "Fuzzy modelling and control of chaotic systems.," *In IEEE International Symposium on circuits and systems, 1996 ISCAS '96*, pp. 209–212.
- Wang, Y., Cheng, L., Hou, Z.G., Tan, M., and Wang, M. (2014), "Leader-Following Output Consensus in a Network of Linear Agents with Communication Noises," *19th IFAC*, 19(1), 1825–1830.
- Xi, J., Yao, Z., Liu, G., and Zhong, Y. (2013), "Swarm stability for high-order linear time-invariant singular multiagent systems," *International Journal of Systems Science*, 0(0), 1–14.
- Yu, L., and Wang, J. (2014), "Distributed output regulation for linear multi-agent systems with communication delays," *International Journal of Systems Science*, pp. 1–17.